

Galilean Transformation

In the adjoining fig the z -axis is perpendicular to the plane of the paper. Then after a time t , the frame s' by a distance $v_x t$, $v_y t$ and $v_z t$ along x , y and z -axes respectively as shown in fig. Then the observations of the event at P taken by both the observers may be seen to be related, referring to before figure given in Galilean Transformation by the equation,

$$\left. \begin{array}{l} x' = x - v_x t \quad (I) \\ y' = y - v_y t \quad (II) \\ z' = z - v_z t \quad (III) \\ t' = t \quad (IV) \end{array} \right\} \quad (2)$$

These are the Galilean Transformations, relating to the observations of position and time made by two observers in two different inertial frames.

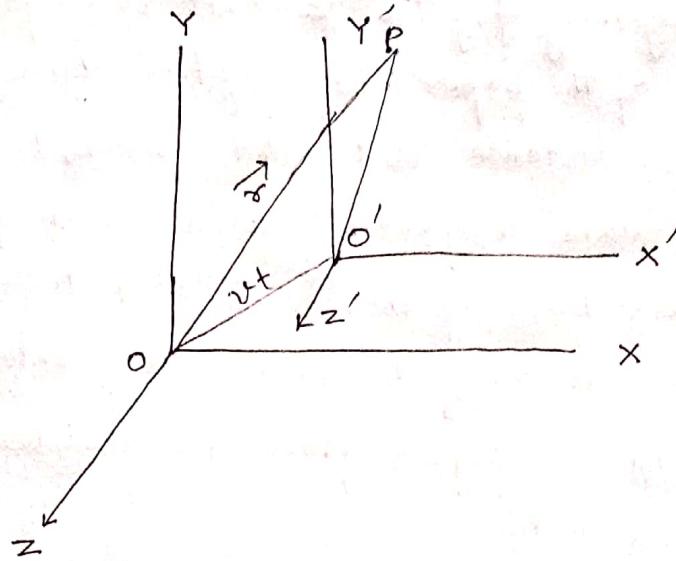
The first three equations may be represented in the form of single equation

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_t t \quad (3)$$

where \mathbf{r} and \mathbf{r}' are the position vectors of the particle P relative to frame s and s' respectively.

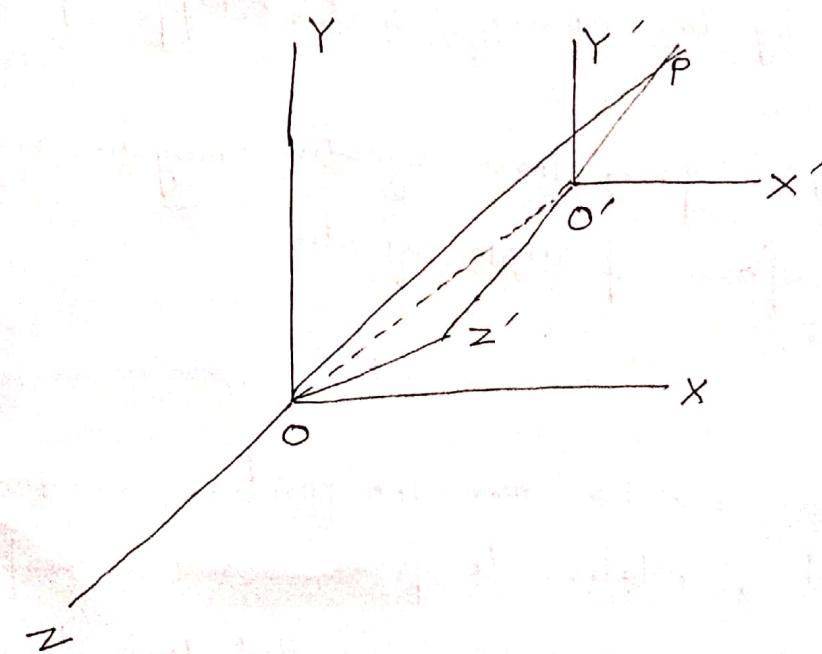
Eqn (1), (2) and (3) are called time dependent Galilean Transformations since they are time dependent

and were obtained by Galileo.



Problem 1. Prove that the Galilean Transformation of a position vector is expressed as $\mathbf{r} = \mathbf{r}_0 + \mathbf{r}' + vt$ where v is the linear velocity of the frame O' as measured by O at $t' = 0$.

Solution:- Consider two frames s and s' the latter



moving with velocity v relative to former. Let O

and O' be the observers situated in s and s' respectively observing the event happening at P . If \mathbf{r} and \mathbf{r}' are the position vectors of the point P at any instant, then we have

$$\mathbf{r} = \mathbf{r}' + \mathbf{R}$$

where \mathbf{R} is the position vector of observer O' relative to O after time t .

If \mathbf{r}_0 is the position vector of the observer O' relative to O at $t = 0$ then from figure we have

$$\begin{aligned}\mathbf{R} &= \overrightarrow{OQ} + \overrightarrow{QO}' \\ &= \mathbf{r}_0 + vt\end{aligned}$$

Since the distance traversed QO' by the observer O' in time t is vt where v is the velocity of the O' relative to O .

Putting the value of R from (2) in one (1) we get

$$\mathbf{r} = \mathbf{r}' + \mathbf{r}_0 + vt$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{r}' + vt' \quad (\text{Since } t = t') \quad (3)$$

Problem 2. Consider two systems s and s' , s' moving with velocity $\mathbf{v} = i v_x + j v_y + k v_z$ relative to s .

If the origins of the two systems coincide at $t = t' = t_0$, find the Galilean transformation transformation equations.

Solution. The system s' is moving relative to s with

velocity v_x , v_y and v_z along +ve directions of x , y and z axes respectively. If the origins of two frames coincides at $t = t' = t_0$ then

the distance traversed by observer O' in s' relative to observer O is

in s at any instant t along axis of $x = v_x(t - t_0)$
the distance traversed by observer O'

in s at any instant t along y axis = $v_y(t - t_0)$

the distance traversed by O' relative to O at any instant t along z -axis = $v_z(t - t_0)$

Thus the Galilean transformation equations are given by

$$x' = x - v_x(t - t_0)$$

$$y' = y - v_y(t - t_0)$$

$$z' = z - v_z(t - t_0)$$

